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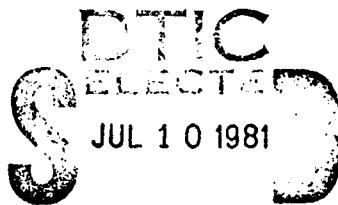
# SPACE CHARGE WAVES IN A CYLINDRICAL WAVEGUIDE WITH ARBITRARY WALL IMPEDANCE

BY HAN S. UHM

RESEARCH AND TECHNOLOGY DEPARTMENT

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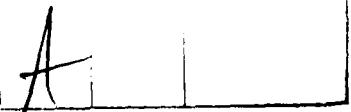
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## FOREWORD



Properties of the space charge waves in a solid relativistic electron beam propagating in a cylindrical waveguide is investigated, including the important influence of arbitrary wall impedance. The stability analysis is carried out within the framework of the linearized Vlasov-Maxwell equations. In order to examine the influence of the axial momentum spread on the stability behavior, it is assumed that all electrons have a Lorentzian distribution in the axial canonical momentum. One of the most important features of the analysis is that, for short axial wavelength perturbations, the eigenfunction can be described by a Bessel function. Moreover, the condition for zero phase velocity of the space charge wave is also obtained, in connection with collective ion acceleration. Space charge wave properties for a dielectric loaded waveguide are also investigated. For appropriate choice of dielectric constant  $\epsilon$  and thickness of the dielectric material, it is shown that a strong mode coupling occurs, exhibiting the growth rate of instability comparable to the beam plasma frequency. The physical mechanism of instability is the Cherenkov radiation. This research was supported by the Independent Research Fund at the Naval Surface Weapons Center.



H. R. RIEDL  
By direction

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I. INTRODUCTION

In recent years, there has been a growing interest in the space charge waves<sup>1-4</sup> in a relativistic electron beam, in connection with collective ion acceleration<sup>3-6</sup> and intense microwave generation.<sup>7-10</sup>

For the most part, previous theoretical analyses<sup>1-4</sup> of the space charge waves are limited and incomplete. However, a complete analysis of the space charge wave is required to optimize the collective ion acceleration and microwave generation. In this regard, in the present article, we develop a unified theory of the space charge wave in a relativistic electron beam, including the important influence of an arbitrary wall impedance on the stability behavior.

Equilibrium and stability properties are calculated for the choice of electron distribution function [Eq. (3)]

$$f_b^0(H, P_\theta, P_z) = \frac{\hat{n}_b \Delta}{2\pi^2 \gamma_b^m} \frac{\delta(H - \omega_b P_\theta - \hat{\gamma} mc^2)}{(P_z - \gamma_b^m \beta_b c)^2 + \Delta^2},$$

where  $H$  is the energy,  $P_\theta$  is the canonical angular momentum,  $P_z$  is the axial canonical momentum, and  $\gamma_b = (1 - \beta_b^2)^{-1/2}$ ,  $\hat{\gamma}$ ,  $\hat{n}_b$ , and  $\Delta$  are constants. The stability analysis of the space charge wave is carried out within the framework of the linearized Vlasov-Maxwell equations, assuming that  $v/\gamma_b \ll 1$ , where  $v$  is Budker's parameter. The formal dispersion relation (20) of the space charge wave for azimuthally symmetric electromagnetic perturbation ( $\partial/\partial\theta = 0$ ) is obtained in Sec. II, including the important influence of finite wall impedance  $Z$ , which is generally an arbitrary function of the eigenfrequency  $\omega$  and axial wavenumber  $k$ . Particularly, we emphasize that Eq. (20) is derived with no a priori assumption that the beam radius  $R_0$  is much less than the wall radius  $R_w$ . In this regard, the dispersion relation in Eq. (20) can be used to investigate properties of the space charge wave for a broad

range of system parameters.

In Sec. III, the resistive wall instability<sup>10</sup> with an arbitrary impedance is investigated in general, assuming that all electrons have the same value of axial canonical momentum ( $\Delta = 0$ ). It is shown that an inductive impedance wall is most unstable. Particularly, the maximum growth rate of the general resistive wall instability occurs at the axial wavenumber  $k$  satisfying  $2k^2R_0^2 = \gamma_b^2(\xi_i^2 - \xi_r^2)$ , where the parameter  $\xi = \xi_r + i\xi_i$  is the root of  $\xi J_1(\xi)/J_0(\xi) = F$ ,  $J_\ell(x)$  is the Bessel function of the first kind of order  $\ell$  and  $F$  is the wave admittance at the beam surface [Eq. (19)].

Properties of the space charge wave in a perfectly conducting waveguide is investigated in Sec. IV, in connection with intense microwave generation<sup>7-10</sup> and collective ion acceleration.<sup>3-6</sup> It is found that the wave admittance  $F$  in a perfectly conducting waveguide is purely capacitive, thereby indicating a stable propagation of the space charge wave. However, for short axial wavelength perturbation satisfying  $kR_0/\gamma_b \gtrsim 10$  [which is typical in the free electron laser application<sup>7,8</sup>], the electrostatic eigenfunction of the space charge wave can be accurately represented by a Bessel function. In the present experiments<sup>4-6</sup> of collective ion acceleration, the phase velocity of the space charge wave is initially required to be zero, in order to trap and accelerate ions. In this regard, in Sec. IV, we also obtain the condition for the phase velocity  $\omega/k = 0$ . That is [Eq. (42)],

$$\frac{4v}{\gamma_b} = (\gamma_b^2 - 1)(\xi_r^2 + k^2R_0^2).$$

We therefore find from Eq. (42) that the typical Budker's parameter for the zero phase velocity is order of unity in a practical range of physical parameters for collective ion acceleration.

Space charge wave properties for a dielectric loaded waveguide are investigated in Sec. V, assuming that the impedance of the dielectric material is purely reactive (i.e., the dielectric constant is perfectly real). In a range of physical parameters, it is shown that the phase velocity of the vacuum dielectric waveguide mode is less than the beam velocity, exhibiting possibilities of a Cherenkov radiation.<sup>9,11</sup> In fact, a strong mode coupling between the vacuum dielectric waveguide mode and the beam streaming mode occurs at the axial wavenumber satisfying  $2k^2 R_0^2 = \gamma_b^2 (\xi_i^2 - \xi_r^2)$ . We therefore conclude that the Cherenkov radiation in a dielectric loaded waveguide is a typical example of the inductive impedance (Sec. III). Several points are noteworthy in the analysis of Sec. V. First, the maximum growth rate of instability is order of the beam plasma frequency. In this regard, the Cherenkov radiation can be utilized to produce high power microwave. However, the growth rate of instability decreases substantially with increasing value of the axial momentum spread. Second, the wavelength of the microwave radiation generated by this instability can be less than a centimeter for a subcentimeter beam radius  $R_0$ . Finally, we note that the growth rate and bandwidth of instability increase rapidly as the surface of dielectric material approaches to the beam surface ( $R_0/R_w \rightarrow 1$ ).

II. THEORETICAL MODEL

The equilibrium configuration consists of a relativistic electron beam column that is infinite in axial extent and aligned parallel to a uniform applied magnetic field  $B_0 \hat{e}_z$ . The electron beam radius is denoted by  $R_0$ , and a finite-impedance wall is located at radius  $r = R_w$ . Cylindrical polar coordinates  $(r, \theta, z)$  are introduced. Moreover, in the present analysis, we assume

$$v/\gamma_b \ll 1 , \quad (1)$$

where  $v = N_b e^2 / mc^2$  is Budker's parameter,

$$N_b = 2\pi \int_0^{R_w} dr \ r \ n_b^0(r) ,$$

is the number of electrons per unit axial length,  $n_b^0(r)$  is the equilibrium electron density,  $c$  is the speed of light in vacuo,  $-e$  and  $m$  are the electron charge and rest mass, respectively, and  $\gamma_b mc^2$  is the characteristic electron energy in the laboratory frame. Consistent with the low-intensity assumption in Eq. (1), we also assume

$$\omega_{pb}^2 / \omega_{cb}^2 \ll 1 , \quad (2)$$

where  $\omega_{pb}^2 = 4\pi e^2 n_b / \gamma_b m$  is the plasma frequency-squared and  $\omega_{cb} = eB_0 / \gamma_b mc$  is the electron cyclotron frequency.

In the present analysis, we investigate stability properties for the choice of equilibrium distribution function

$$f_b^0(H, P_\theta, P_z) = \frac{\hat{n}_b \Delta}{2\pi \gamma_b^m} \frac{\delta(H - \omega_b P_\theta - \gamma_b mc^2)}{(P_z - \gamma_b m \omega_b c)^2 + \Delta^2} , \quad (3)$$

where  $H = (m^2 c^4 + c^2 p_\theta^2)^{1/2} - e\phi_0(r)$  is the total energy,  $P_\theta = r[p_\theta - (e/2c)rB_0]$  is the canonical angular momentum,  $P_z = p_z - (e/c)A_z^S(r)$  is the axial canonical momentum,  $\phi_0(r)$  and  $A_z^S(r)$  are the equilibrium electrostatic

and axial component of vector potentials, respectively,  $\gamma_b = (1 - \beta_b^2)^{-1/2}$ ,  $\hat{y}$ ,  $\hat{n}_b$ , and  $\Delta$  are constants.

In the subsequent perturbation analysis, use is made of the linearized Vlasov-Maxwell equations for azimuthally symmetric perturbations ( $\partial/\partial\theta = 0$ ) about a solid electron beam described by Eq. (3). We adopt a normal-mode approach in which all perturbations are assumed to vary according to

$$\delta\psi(\mathbf{x}, t) = \hat{\psi}(\mathbf{r}) \exp[i(kz - \omega t)] ,$$

where  $\text{Im}\omega > 0$ . Here,  $\omega$  is the complex eigenfrequency and  $k$  is the axial wavenumber. The Maxwell equations for the perturbed electric and magnetic field amplitudes can be expressed as

$$\hat{\nabla} \times \hat{\mathbf{E}}(\mathbf{x}) = i(\omega/c) \hat{\mathbf{B}}(\mathbf{x}) , \quad (4)$$

$$\hat{\nabla} \times (1/\mu) \hat{\mathbf{B}}(\mathbf{x}) = (4\pi/c) \hat{\mathbf{J}}(\mathbf{x}) - i(\omega/c) \epsilon \hat{\mathbf{E}}(\mathbf{x}) ,$$

where  $\epsilon$  and  $\mu$  are the dielectric constant and permeability, respectively,  $\hat{\mathbf{E}}(\mathbf{x})$  and  $\hat{\mathbf{B}}(\mathbf{x})$  are the perturbed electric and magnetic fields, and

$$\hat{\mathbf{J}}(\mathbf{x}) = -e \int d^3 p \ \hat{\mathbf{v}} \ f_b(\mathbf{x}, \mathbf{p}) , \quad (5)$$

is the perturbed current density. Note that  $\epsilon = \mu = 1$  in vacuo.

In Eq. (5),

$$\hat{f}_b(\mathbf{x}, \mathbf{p}) = e \int_{-\infty}^0 d\tau \exp(-i\omega\tau) \left( \hat{f}_b(\mathbf{x}') + \frac{\hat{\mathbf{v}}' \times \hat{\mathbf{B}}(\mathbf{x}')}{c} \right) \cdot \frac{\partial}{\partial \mathbf{p}'} f_b^0 , \quad (6)$$

is the perturbed distribution function,  $\tau = t' - t$ , and the particle trajectories  $\mathbf{x}'(t')$  and  $\mathbf{p}'(t')$  satisfy  $d\mathbf{x}'/dt' = \hat{\mathbf{v}}'$  and  $d\mathbf{p}'/dt' = -e\hat{\mathbf{v}}' \times \hat{\mathbf{B}}_0^s/c$ , with "initial" conditions  $\mathbf{x}'(t' = t) = \mathbf{x}$  and  $\mathbf{v}'(t' = t) = \hat{\mathbf{v}}$ .

In general, the permeability  $\mu$  of a dielectric material even in the wall differs from unity by only a few parts in  $10^5$ . Therefore, we approximate  $\mu = 1$  in the remainder of this paper. Making use of Eq. (4), it is straightforward to show that

$$\hat{E}_r(r) = (kc/\omega\epsilon)\hat{B}_\theta(r) , \quad (7)$$

$$\hat{B}_\theta(r) = i[\omega\epsilon/c(\omega^2\epsilon/c^2 - k^2)][\partial\hat{E}_z(r)/\partial r] ,$$

and

$$\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial}{\partial r} + \frac{\omega^2}{c^2}\epsilon - k^2\right)\hat{E}_z(r) = 4\pi ik\left(\hat{\rho}(r) - \frac{\omega}{c^2k}\hat{j}_z(r)\right) , \quad (8)$$

where  $\hat{B}_\theta$  is the azimuthal component of the perturbed magnetic field, and  $\hat{E}_r$  and  $\hat{E}_z$  are the radial and axial components, respectively, of the perturbed electric field,  $\hat{\rho}(r)$  is the perturbed charge density and  $\hat{j}_z(r)$  is the axial component of the perturbed current density.

To lowest order, the axial motion of the particle orbit is free-streaming

$$z' = z + \frac{p_z}{\gamma m} (t' - t) . \quad (9)$$

Moreover, within the context of Eq. (2), we neglect the terms proportional to  $\hat{E}_\perp(r)$  on the right-hand side of Eq. (6), where  $\hat{E}_\perp(r)$  is the transverse component of the perturbed electric field. Finally we assume a slow rotational equilibrium characterized by

$$\omega_b \approx \omega_b^- = \frac{\omega_{cb}}{2} \left(1 - \left(1 - \frac{2\omega_{pb}^2}{\gamma_b^2 \omega_{cb}^2}\right)^{1/2}\right) , \quad (10)$$

thereby approximating<sup>12</sup>

$$r' \approx r , \quad (11)$$

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in the arguments of the perturbation amplitudes on the right-hand side of Eq. (6).

Substituting Eqs. (9) and (11) into Eq. (6), we obtain the perturbed distribution function

$$\hat{f}_b(r, p) = \frac{e i}{\omega - k p_z / \gamma_m} \hat{E}_z(r) \frac{\partial f_b^0}{\partial p_z} . \quad (12)$$

Carrying out the momentum integration with Eqs. (3) and (12), the differential equation (8) can be expressed as

$$\left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + T^2 \right) \hat{E}_z(r) = 0 , \quad 0 \leq r < R_0 , \quad (13)$$

$$\left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + p^2 \right) \hat{E}_z(r) = 0 , \quad R_0 < r < R_w ,$$

where

$$T^2 = \left( k^2 - \frac{\omega^2}{c^2} \right) \left[ \frac{\omega_{pb}^2}{\gamma_b^2 (\omega - k \beta_b c + i k \Delta / \gamma_b^3)^2} - 1 \right] , \quad (14)$$

$$p^2 = \frac{\omega^2}{c^2} - k^2 ,$$

and the beam radius  $R_0$  is defined by

$$R_0^2 = 2c^2(\gamma - \gamma_b) / \gamma_b (\omega_b \omega_{cb} - \omega_b^2 - \omega_{pb}^2 / 2\gamma_b^2) . \quad (15)$$

For the convenience in the future analysis, we define the wave impedance  $Z(\omega, k)$  of the wall as

$$Z(\omega, k) = -i(\omega R_w / c) \frac{\hat{E}_z(R_w)}{\hat{B}_\theta(R_w)} , \quad (16)$$

where  $R_w$  is the radius of the finite impedance wall. Evidently, the solutions to Eq. (13) are given by

$$\hat{E}_z(r) = \begin{cases} AJ_0(Tr) , & 0 \leq r < R_0 , \\ BJ_0(pr) + CN_0(pr) , & R_0 < r \leq R_w , \end{cases} \quad (17)$$

where  $J_\ell(x)$  and  $N_\ell(x)$  are Bessel functions of the first and second kind, respectively, of order  $\ell$ . Making use of Eqs. (7) and (16), and the boundary conditions of  $\hat{E}_z(r)$  at  $r = R_0$ , we obtain the dispersion relation,

$$\frac{\xi J_1(\xi)}{J_0(\xi)} = F(\omega, k) , \quad (18)$$

where  $F(\omega, k)$  is the wave admittance at the beam surface defined by

$$F(\omega, k) = pR_0 \frac{J_1(pR_0) + g(pR_w)N_1(pR_0)}{J_0(pR_0) + g(pR_w)N_0(pR_0)} , \quad (19)$$

and

$$\xi = TR_0 . \quad (20)$$

In Eq. (19),

$$g(x) = \frac{J_1(x)}{xN_0(x) - zN_1(x)} \left( z - \frac{xJ_0(x)}{J_1(x)} \right) , \quad (21)$$

and  $p$  is defined in Eq. (14). In the remainder of this article, we make use of Eq. (18) to investigate properties of the space charge wave for various values of wall impedance  $Z$  in Eq. (16).

III. INFLUENCE OF WAVE ADMITTANCE ON THE SPACE CHARGE WAVES

For convenience in the subsequent analysis, we denote the root of Eq. (18) by

$$\xi = \xi_r(\omega, k) + i\xi_i(\omega, k) , \quad (22)$$

for a specified value of wave admittance

$$F = f \exp(i\phi) = F_r + iF_i , \quad (23)$$

where  $\xi_r$  and  $\xi_i$  are the real and imaginary parts of  $\xi$ , and  $f$  and  $\phi$  are the magnitude and phase angle, respectively, of the general wave admittance  $F$  at the beam surface. From Eq. (18) with Eq. (23), we note

$$\frac{\xi^* J_1(\xi^*)}{J_0(\xi^*)} = f \exp(-i\phi) , \quad (24)$$

where  $\xi^*$  is the complex conjugate of  $\xi$  defined by  $\xi^* = \xi_r - i\xi_i$ .

Shown in Fig. 1 are contours of constant phase angle  $\phi$  and modulus  $f$  of the function  $F = \xi J_1(\xi)/J_0(\xi)$  in the complex plane  $\xi = \xi_r + i\xi_i$ . We note from Fig. 1 that the root  $\xi$  of Eq. (18) approaches zero or  $\alpha_{0n}$  as the magnitude of the wave admittance is reduced to zero,  $f \rightarrow 0$ .

On the other hand,  $\xi$  approaches  $\beta_{0n}$  as  $f$  increases to infinity.

Here  $\alpha_{0n}$  and  $\beta_{0n}$  are the  $n$ th root of  $J_1(\alpha_{0n}) = 0$  and  $J_0(\beta_{0n}) = 0$ , respectively. For specified values of  $f$  and  $\phi$ , we note that the root  $\xi = \xi_r + i\xi_i$  can be determined from Fig. 1.

Within the context of Eq. (1), it is very useful in the subsequent analysis to note that

$$|\omega - k\beta_b c| \ll kc . \quad (25)$$

In the remainder of this section, we investigate the resistive wall instability<sup>10</sup> in general, assuming that all electrons have the same value of axial canonical momentum, i.e.,

$$\Delta = 0 . \quad (26)$$

Making use of Eqs. (14), (20), and (26), we obtain,

$$\Omega_i^2 = \frac{1}{2[(y + \delta)^2 + 1]} \left\{ |y| [(y + \delta)^2 + 1]^{1/2} - y(y + \delta) \right\} , \quad (27)$$

for the resistive wall instability in general. In Eq. (27),  $\Omega_i$  is the normalized growth rate defined by

$$\Omega_i = \frac{\text{Im}(\omega - k\beta_b c)}{\omega_{pb}/\gamma_b} , \quad (28)$$

and the parameters  $y$  and  $\delta$  are given by

$$y = k^2 R_0^2 / 2\gamma_b^2 \xi_r \xi_i , \quad (29)$$

and

$$\delta = (\xi_r^2 - \xi_i^2) / 2\xi_r \xi_i , \quad (30)$$

respectively. In obtaining Eq. (27), use has been made of Eq. (25).

(a) Inductive impedance with  $\xi_i^2 > \xi_r^2$ . Shown in Fig. 2 are plots of the normalized growth rate  $\Omega_i$  versus  $y$  obtained from Eq. (27) for  $\xi_i^2 > \xi_r^2$  and several different values of  $\delta$ . As shown in Fig. 2,  $\Omega_i^m$  is the maximum growth rate for a given value of parameter  $\delta$ , and  $y_m^I$  is the corresponding value of  $y$ . Obviously from Fig. 2, the maximum growth rate increases rapidly with the increasing value of  $|\delta|$  in the inductive impedance case. In Fig. 3, we plot the parameter  $y_m^I$  corresponding to the maximum growth rate versus  $\delta$  (solid line) obtained from Eq. (27) and  $y_m^I = -\delta$  (dashed line). Evidently, we note

from Fig. 3 that the maximum growth rate  $\Omega_i^m$  and the corresponding  $y_m^I$  can be approximated by

$$\Omega_i^m \approx (|\delta|/2)^{1/2}, \quad (31)$$

and

$$y_m^I \approx -\delta, \quad (32)$$

for  $|\delta| \geq 1$ . Substituting Eqs. (29) and (30) into Eq. (32), we conclude for the inductive impedance that the maximum growth rate of the resistive wall instability occurs at the axial wavenumber  $k$  satisfying  $k^2 R_0^2 = \gamma_b^2 (\xi_i^2 - \xi_r^2)$  for  $|\delta| \geq 1$ .

(b) Capacitive impedance with  $\xi_r^2 > \xi_i^2$ . Presented in Fig. 4 are plots of the normalized growth rate  $\Omega_i^m$  versus  $y$  obtained from Eq. (27) for  $\xi_r^2 > \xi_i^2$  and several values of parameter  $\delta$ . As shown in Fig. 4, we again define the maximum growth rate  $\Omega_i^m$  and corresponding  $y_m^c$  for a given value of  $\delta$ . In order to illustrate the stability dependence on the parameter  $\delta$ , in Fig. 5, we plot the maximum growth rate  $\Omega_i^m$  and corresponding  $y_m^c$  versus  $\delta$  obtained from Eq. (27) for  $\xi_r^2 > \xi_i^2$ . The growth rate increases rapidly with the decreasing value of  $|\delta|$  for the capacitive impedance case.

(c) Perfectly resistive with  $\xi_r^2 = \xi_i^2$ . For the perfectly resistive impedance case characterized by

$$\xi_r^2 = \xi_i^2, \quad (33)$$

Eq. (27) can be simplified as

$$\Omega_i^2 = \frac{1}{2(y^2 + 1)} [ |y| (y^2 + 1)^{1/2} - y^2 ], \quad (34)$$

which gives the maximum growth rate

$$\Omega_i^m = 0.3536, \quad (35)$$

at  $y = 0.577$ . We therefore conclude that the inductive impedance case is most unstable. In the following sections, we investigate properties of the space charge waves in perfectly capacitive and inductive impedances.

IV. SPACE CHARGE WAVES IN A PERFECTLY CONDUCTING WAVEGUIDE

In this section, we investigate properties of the space charge waves in a relativistic electron beam assuming that the wall impedance is zero, i.e.,

$$Z = 0, \quad (36)$$

and that all electrons have the same value of axial canonical momentum ( $\Delta = 0$ ). Substituting Eqs. (21) and (36) into Eq. (19) and defining

$$q^2 = (k^2 - \omega^2/c^2)R_0^2, \quad (37)$$

we obtain

$$F(q) = q \frac{I_1(q)K_0(R_w q/R_0) + I_0(R_w q/R_0)K_1(q)}{I_0(R_w q/R_0)K_0(q) - I_0(q)K_0(R_w q/R_0)}, \quad (38)$$

where  $I_\ell(x)$  and  $K_\ell(x)$  are the modified Bessel functions of the first and second kind, respectively, of order  $\ell$ . For a real value of  $q$ , the wave admittance  $F(q)$  in Eq. (38) is a positive real value ( $F_i = 0$ ), thereby giving the phase angle  $\phi = 0$  in Eq. (23). In this regard, the space charge wave admittance  $F$  in a perfectly conducting waveguide is a perfectly capacitive, corresponding to the horizontal line  $\xi_i = 0$  in Fig. 1. Without further analysis, we therefore conclude from Fig. 4 that the space charge mode in a perfectly conducting waveguide is stable [ $\delta = \infty$  in Eq. (27)].

Figure 6 shows plots of the admittance  $F = F_r$  versus  $\zeta = \xi_r$  (dashed curves) obtained from Eq. (18) and  $F_r$  versus  $q$  (solid curves) obtained from Eq. (38) for  $R_0/R_w = 0.1, 0.5$  and  $0.9$ . The horizontal scale in Fig. 6 represents both  $\xi_r$  and  $q$ . In Fig. 6,  $n = 1, 2$ , and  $3$  denote the radial mode number of the space charge wave. Note that

for specified values of  $q$  and  $R_w/R_0$ , the parameter  $\xi_r$  is determined from Fig. 6. Shown in Fig. 7 is plot of  $\xi_r$  versus  $q$  determined from Fig. 6 for  $R_0/R_w = 0.5$ . Obviously, the root  $\xi_r$  approaches  $\beta_{0n}$  as the parameter  $q$  increases to infinity. This is similar to Fig. 1, since the wave admittance  $F_r$  is monotonically increasing with the increasing value of  $q$ .

In order to illustrate the influence of the parameter  $q$  on the space charge mode, we present in Fig. 8 plots of the eigenfunction  $\hat{E}_z(r)$  versus  $r/R_0$  obtained from Eq. (17) for (a)  $n = 1$ , (b)  $n = 2$ ,  $R_0/R_w = 0.5$  and several values of  $q$ . The perturbed axial electric field  $\hat{E}_z(r)$  in Fig. 8 is normalized by

$$\frac{1}{R_0^2} \int_0^{R_w} dr r \hat{E}_z(r) = 1.$$

It is obvious from Fig. 8 that the axial electric field  $\hat{E}_z(r)$  in Eq. (17) can be approximated by

$$\hat{E}_z(r) \approx \begin{cases} AJ_0(\beta_{0n}r/R_0) & , 0 \leq r < R_0 , \\ 0 & , \text{otherwise} , \end{cases} \quad (39)$$

for  $q \gtrsim 10$ . We therefore conclude that inside the beam, the electrostatic eigenfunction of the large axial wavenumber perturbations is represented by a Bessel function. For example, Eq. (39) is an excellent approximation for the electrostatic eigenfunction of the free electron laser instability.<sup>7,8</sup>

Making use of Eq. (20) and (37), Eq. (14) can be expressed as

$$(\omega - k\beta_b c)^2 = \frac{\omega_{pb}^2 q^2}{\gamma_b^2 (\xi_r^2 + q^2)} , \quad (40)$$

where the root  $\xi_r(q)$  is determined from Fig. 7 for given values of  $R_0/R_w$  and  $n$ , and use has been made of  $\Delta = 0$ . The eigenfrequency  $\omega$  and axial wavenumber  $k$  are obtained from the simultaneous solution of

Eqs. (37) and (40). Shown in Fig. 9 is a plot of the dispersion curve in the  $(\omega, k)$  parameter space for  $n = 1$ ,  $R_0/R_w = 0.5$ ,  $\gamma_b = 3$ , and  $v/\gamma_b = \omega_{pb}^2 R_0^2 / 4c^2 = 0.1$ . Simultaneously solving Eqs. (37) and (40) for  $q$  and  $\xi_r(q)$  give two distinct dispersion curves. The fast wave mode in Fig. 9 corresponds to the phase velocity  $v_{ph} = \omega/k > \beta_b c$  and the slow wave mode corresponds to  $v_{ph} < \beta_b c$ .

After some straightforward algebraic manipulation with the definition  $v/\gamma_b = \omega_{pb}^2 R_0^2 / 4c^2$  we can rewrite Eq. (40) by

$$\left( \frac{\omega}{kc} - \beta_b \right)^2 = \frac{4v}{\gamma_b^3} \frac{1 - \omega^2/k^2 c^2}{\xi_r^2 + (1 - \omega^2/k^2 c^2) k^2 R_0^2}, \quad (41)$$

which determines the phase velocity  $v_{ph} = \omega/k$  of the space charge wave in terms of the normalized axial wavenumber  $kR_0$ , the Budker's parameter  $v$  and energy  $\gamma_b$  of the electron beam. In recent years, there has been a considerable increase in interest in collective ion acceleration by a slow space-charge wave<sup>3-6</sup> in a relativistic electron beam. However, the phase velocity of the space charge wave is initially required to be zero, in order to trap and accelerate ions. In this regard, we derive the condition for  $v_{ph} = \omega/k = 0$  from Eq. (41). That is,

$$\frac{4v}{\gamma_b} = (\gamma_b^2 - 1) [\xi_r^2 (kR_0) + k^2 R_0^2], \quad (42)$$

where the root  $\xi_r$  is determined in terms of the normalized axial wavenumber  $kR_0$ . For given values of  $R_0/R_w$  and  $n$ , the root  $\xi_r$  required in Eq. (42) can be found from Fig. 7 where the horizontal scale  $q$  is replaced by  $kR_0$ .

Figure 10 is plots of  $\xi_r$  versus  $R_0/R_w$  for  $n = 1$  and several values of  $kR_0$ . The root  $\xi_r$  is monotonically increasing to  $\xi_{01} = 2.4$  as the ratio  $R_0/R_w$  increases from zero to unity. Making use of Fig. 10,

the necessary value of Budker's parameter for the zero phase velocity of space charge wave is found from Eq. (42). For example, for  $kR_0 = 1$ ,  $R_0/R_w = 0.5$  and  $\gamma_b = 1.2$ , we find  $4v/\gamma_b = 1.46$  for  $v_{ph} = \omega/k = 0$ . Typical Budker's parameter for the zero phase velocity is order of unity in a practical range of physical parameters for collective ion acceleration. Finally, for a small phase velocity satisfying  $|\omega/kc| \ll \beta_b$ , Eq. (41) can be approximated by

$$\frac{4v}{\gamma_b} \approx (\gamma_b^2 - 1) \left[ \xi_r^2 + \left( \frac{\omega R_0}{c} \right) \left( \frac{kc}{\omega} \right)^2 \right], \quad (43)$$

which is identical to Eq. (42). However, from Eq. (43), the Budker's parameter is determined in terms of the oscillation frequency  $\omega$  and the phase velocity  $v_{ph} = \omega/k$  required to ion accelerations.

V. SPACE CHARGE WAVES FOR A DIELECTRIC LOADED WAVEGUIDE

In this section, we investigate properties of the space charge waves for a solid electron beam in a dielectric loaded waveguide.

The solid electron beam with radius  $R_0$  propagates through a cylindrical waveguide loaded with dielectric material in the range  $R_w < r < R_c$ .

A grounded cylindrical conducting wall is located at radius  $R_c$ .

We approximate the permeability of a dielectric material  $\mu = 1$ . In this regard, the perturbed axial electric field  $\hat{E}_z(r)$  and azimuthal magnetic field  $\hat{B}_\theta(r)$  are continuous across the dielectric boundary at  $r = R_w$ . From Eq. (8), we obtain

$$\left( \frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} + p_2^2 \right) \hat{E}_z(r) = 0, \quad (44)$$

inside the dielectric material ( $R_w \leq r \leq R_c$ ). Here  $p_2$  is defined by

$$p_2^2 = \omega^2 \epsilon / c^2 - k^2. \quad (45)$$

The solution to Eq. (44) can be expressed as

$$\hat{E}_z(r) = A [J_0(p_2 r) - J_0(p_2 R_c) N_1(p_2 r) / N_0(p_2 R_c)], \quad (46)$$

where  $A$  is an arbitrary constant. Substituting Eq. (46) into Eq. (7), and making use of the boundary conditions at  $r = R_w$  and Eq. (16), we obtain the impedance

$$Z = \frac{\eta}{\epsilon} \frac{J_0(\eta) N_0(\eta R_c / R_w) - J_0(\eta R_c / R_w) N_0(\eta)}{J_1(\eta) N_0(\eta R_c / R_w) - J_0(\eta R_c / R_w) N_1(\eta)}, \quad (47)$$

at the surface of dielectric material ( $r = R_w$ ). In Eq. (47), the parameter  $\eta$  is defined by

$$\eta^2 = p_2^2 R_w^2 = R_w^2 (\omega^2 \epsilon / c^2 - k^2). \quad (48)$$

Equation (18), when combined with Eqs. (19), (21), and (47), yields a closed dispersion relation for the space charge wave mode in a dielectric loaded waveguide.

Making use of Eqs. (14), (19), and (20), it can be shown that in the vacuum dielectric waveguide characterized by  $\omega_{pb}^2 = 0$  in Eq. (14), the dispersion relation in Eq. (18) is simplified as

$$g(pR_w) = 0 , \quad (49)$$

where  $p = (\omega^2/c^2 - k^2)^{1/2}$ . Substituting Eqs. (21) and (47) into Eq. (49), we obtain the vacuum transverse magnetic (TM) mode dispersion relation,

$$\frac{pR_w J_0(pR_w)}{J_1(pR_w)} = \frac{\eta}{\epsilon} \frac{J_0(\eta)N_0(\eta R_c/R_w) - J_0(\eta R_c/R_w)N_0(\eta)}{J_1(\eta)N_0(\eta R_c/R_w) - J_0(\eta R_c/R_w)N_1(\eta)} , \quad (50)$$

where the parameter  $pR_w$  is defined by

$$p^2 R_w^2 = R_w^2 (\omega^2/c^2 - k^2) . \quad (51)$$

It is instructive to examine Eq. (50) in the limit  $\epsilon \rightarrow 1$ .

Making use of  $pR_w = \eta$ , we obtain

$$J_0(\eta R_c/R_w) = 0 , \quad (52)$$

from Eq. (50) for  $\epsilon = 1$ . Equation (52) gives the familiar vacuum TM mode dispersion relation,

$$\frac{\omega^2}{c^2} - k^2 = \frac{\beta_{0n}^2}{R_c^2} , \quad (53)$$

in a perfectly conducting waveguide. Moreover, we can show that in the limit of both  $R_w \rightarrow R_c$  and  $R_w \rightarrow 0$ , Eq. (50) gives the dispersion relation in Eq. (53) and,

$$\frac{\omega^2}{c^2} \epsilon - k^2 = \frac{\beta_{0n}^2}{R_c^2} , \quad (54)$$

respectively. Note that the case  $R_w \rightarrow 0$  corresponds to a completely filled dielectric waveguide.

For given values of the dielectric constant  $\epsilon$  and the ratio  $R_w/R_c$ , the parameter  $pR_w$  is determined from Eq. (50) in terms of  $\eta$ . The oscillation frequency  $\omega$  and axial wavenumber  $k$  in a vacuum dielectric loaded waveguide are obtained from the simultaneous solution of Eqs. (48) and (51) for specified  $\eta$  and  $pR_w$ . Figure 11 is plots of the vacuum TM mode dispersion relation in the  $(\omega, k)$  parameter space for (a)  $R_w/R_c = 0.8$  and several values of the dielectric constant  $\epsilon$ , and (b)  $\epsilon = 4$  and several values of the ratio  $R_w/R_c$ . We remind the reader that the thickness of dielectric material increases from zero to  $R_c$  as the ratio  $R_w/R_c$  decreases from unity to zero. All plots in Fig. 11 correspond to the first radial mode number ( $n = 1$ ). The dispersion curve for  $R_w/R_c = 0$  in Fig. 11(b) represents the dispersion relation of a completely filled dielectric waveguide [Eq. (54)]. On the other hand, the curves for  $\epsilon = 1$  in Fig. 11(a) and for  $p_w/R_c = 1$  in Fig. 11(b) correspond to the ordinary dispersion relation in Eq. (53) where the phase velocity  $V_{ph} = \omega/k$  is always faster than the speed of light ( $\omega/k > c$ ). However, the phase velocity of the dispersion relation in a dielectric loaded waveguide is sometimes less than the speed of light ( $\omega/k < c$ ). For example, for  $R_w/R_c = 0.8$  and  $\epsilon = 4$  in Fig. 11(a), we find  $\omega/k > c$  for  $kR_c < 3.3$  and  $\omega/k < c$  for  $kR_c > 3.3$ .

It is evident from Eq. (14) and (20) that in the vacuum dielectric waveguide ( $\omega_{pb}^2 = 0$ ), we can equivalently express the combination of Eqs. (48), (50), and (51) as,

$$\frac{\omega^2}{c^2} - k^2 = \xi^2(\omega, k)/R_0^2, \quad (55)$$

where  $\xi(\omega, k)$  is determined from Eq. (18) in terms of the wave admittance  $F$  at  $r = R_0$ . Equation (55) is a compact form of the vacuum TM mode dispersion relation in a dielectric loaded waveguide. Shown in Fig. 12 is a plot of  $\omega = (k^2 c^2 + \xi_{0n}^2 c^2 / R_c^2)^{1/2}$  versus  $k$  corresponding to a perfectly conducting waveguide and  $\omega = (k^2 c^2 + \xi^2 c^2 / R_0^2)^{1/2}$  versus  $k$  corresponding to a dielectric loaded waveguide. The straight line  $\omega = k\beta_b c$  represents the free-streaming mode. In a range of physical parameters, the mode  $\omega = k\beta_b c$  intersects  $\omega = (k^2 c^2 + \xi^2 c^2 / R_0^2)^{1/2}$  at  $k = k_p$ , indicating a possible mode coupling. In fact, for  $k > k_p$  in Fig. 12, the phase velocity of the vacuum dielectric mode is less than the beam velocity. In this regard, we expect a strong Cherenkov radiation<sup>10,11</sup> near the intersection point of these two modes.

In order to investigate stability properties of the space charge wave in a dielectric loaded waveguide, it is necessary to numerically solve Eq. (18) with no a priori assumption that the beam is very tenuous. However, use is made of the fact that the Doppler-shifted eigenfrequency  $\omega - k\beta_b c$  is well removed from the free-streaming mode, i.e.,  $|\omega - k\beta_b c| \ll k\beta_b c$ . Evaluating the parameter  $\eta$  in Eq. (47) and the wave admittance  $F$  in Eq. (19) at  $\omega = \omega_0 = k\beta_b c$ , the dispersion relation in Eq. (18) can be approximated by

$$\frac{(\xi_0 + \delta\xi)J_1(\xi_0 + \delta\xi)}{J_0(\xi_0 + \delta\xi)} = F_0 + (\partial F / \partial \omega)_{\omega_0} (\omega - k\beta_b c) , \quad (56)$$

where

$$F_0 = F(\omega_0, k) , \quad (57)$$

the parameter  $\xi_0 = \xi(\omega_0)$  is defined by

$$\frac{\xi_0 J_1(\xi_0)}{J_0(\xi_0)} = F_0 , \quad (58)$$

and

$$\delta\xi = \xi - \xi_0 . \quad (59)$$

Taylor expanding the left-hand side of Eq. (56) about  $\xi = \xi_0$  and making use of Eq. (58), we approximate Eq. (56) by

$$\xi^2 = \xi_0^2 \left( 1 + \frac{2}{\xi_0^2 + F_0^2} (\partial F / \partial \omega)_{\omega_0} (\omega - k\beta_b c) \right) , \quad (60)$$

where use has been made of the assumption that the term proportional to  $(\partial F / \partial \omega)_{\omega_0}$  in Eq. (60) is much less than unity. Substituting Eqs. (14) and (20) into Eq. (60), we finally have the dispersion relation,

$$\begin{aligned} & \left[ \frac{\omega_{pb}^2}{\gamma_b^2(\omega - k\beta_b c + ik\Delta/\gamma_b^3 m)^2} - 1 \right] \left( k^2 - \frac{\omega^2}{c^2} \right) R_0^2 \\ & = \xi_0^2 \left[ 1 + \frac{2}{\xi_0^2 + F_0^2} (\partial F / \partial \omega)_{\omega_0} (\omega - k\beta_b c) \right] , \end{aligned} \quad (61)$$

for the space charge wave in a dielectric loaded waveguide.

Defining the normalized Doppler-shifted eigenfrequency  $\Omega$  by

$$\Omega = \frac{\omega - k\beta_b c}{\omega_{pb}/\gamma_b} , \quad (62)$$

the dispersion relation in Eq. (61) is numerically investigated for a broad range of physical parameters. For present purposes, to illustrate the mode coupling of the space charge wave ( $\omega = k\beta_b c$ ) with the vacuum dielectric mode, shown in Fig. 13 are plots of (a)  $F_0$  (solid curve) and  $\xi_0^2$  (dashed curve), (b) the normalized growth rate  $\Omega_1 = 1m\Omega$  and (c) Doppler-shifted real oscillation frequency  $\Omega_r = \text{Re}\Omega$  versus  $kR_0$  obtained from Eqs. (57), (58), and (61), for  $\gamma_b = 2$ ,  $\epsilon = 8$ ,  $R_0/R_w = 0.8$ ,  $R_w/R_c = 0.8$ , and  $v = 0.0025$ . The real oscillation frequency in Fig. 13(c) is obtained for zero axial momentum spread ( $\Delta = 0$ ). In Fig. 13(c), the solid curve represents the unstable mode and the dashed curves correspond to stable oscillations. Several points are noteworthy in

Fig. 13. First, the maximum growth rate for instability occurs at  $kR_0 = \gamma_b(-\xi_0^2)^{1/2}$  corresponding to the mode coupling point  $k_p$  in Fig. 12. This is consistent with the inductive impedance in Sec. III. For example, in Fig. 13, the maximum coupling occurs at  $kR_0 = 2$  and  $\xi_0^2 = -1$ . Second, the maximum growth rate is order of the beam plasma frequency, indicating a strong instability. In this regard, this instability can be utilized to generate high power microwave. Third, wavelength of the microwave radiation generated by this instability can be less than a centimeter for a subcentimeter beam radius. Fourth, from Fig. 13(c), we note that the Doppler-shifted real frequency  $\Omega_r$  for instability is negative, thereby implying that the phase velocity of unstable mode is less than the beam velocity. We therefore conclude that the instability mechanism is a typical Cherenkov radiation.<sup>11</sup> Finally, the growth rate and bandwidth of instability decrease with increasing value of the axial momentum spread.

The dependence of stability properties on the ratio  $R_0/R_w$  is further illustrated in Fig. 14, where the normalized growth rate  $\Omega_1 = 1m\Omega$  is plotted versus  $kR_0$  for  $\Delta = 0$ , several values of  $R_0/R_w$ , and parameters otherwise identical to Fig. 13. Obviously from Fig. 14, we note that the growth rate and bandwidth of instability increase rapidly as the surface of dielectric material approaches to the beam surface ( $R_0/R_w \rightarrow 1$ ) for a given beam radius. Shown in Fig. 15 is plots of the normalized growth rate versus  $kR_0$  obtained from Eq. (61) for  $\gamma_b = 1.1547$ ,  $\epsilon = 25$ , and parameters otherwise identical to Fig. 13(b). Even for a moderate beam energy, the growth rate of instability is also order of the beam plasma frequency. However, the maximum growth rate of instability is rapidly decreasing with increasing value of the axial momentum spread.

Finally, we conclude this section by pointing out that from Figs. 13(a) and (b), the maximum coupling of instability occurs when the wave admittance  $F_0$  is negative [the phase angle  $\phi = \pi$  in Eq. (23)]. In this regard, the root  $\xi$  can be a pure imaginary value (the  $\xi_r = 0$  vertical line in Fig. 1). We therefore emphasize that the wave admittance  $F$  in a dielectric loaded waveguide can be perfectly inductive in a range of the axial wavenumber  $k$  corresponding to instability.

VI. CONCLUSIONS

In this paper we have investigated properties of the space charge wave in a solid relativistic electron beam propagating in a cylindrical waveguide with an arbitrary impedance  $Z$ . The perturbation analysis was carried out within the framework of the linearized Vlasov-Maxwell equations, assuming that  $v/\gamma_b \ll 1$ . The formal dispersion relation of the space charge wave for azimuthally symmetric electromagnetic perturbations ( $\partial/\partial\theta = 0$ ) was carried out in Sec. II, including the important influence of finite wall impedance  $Z$ . In Sec. III, the resistive wall instability with an arbitrary impedance was investigated, showing that an inductive impedance wall is most unstable. Particularly, the maximum growth rate of the general resistive wall instability occurs at the axial wavenumber  $k$  satisfying  $2k^2 R_0^2 = \gamma_b^2(\xi_i^2 - \xi_r^2)$ . Properties of the space charge wave in a perfectly conducting waveguide was investigated in Sec. IV. It was found that the space charge wave admittance in a perfectly conducting waveguide is purely capacitive, thereby indicating a stable propagation of the electromagnetic wave. Moreover, we obtained the condition for the zero phase velocity ( $\omega/k = 0$ ), in connection with collective ion acceleration. Furthermore, it was also shown in Sec. IV that for short axial wavelength perturbations ( $kR_0/\gamma_b \gtrsim 10$ ), the eigenfunction can be represented by a Bessel function. Space charge wave properties for a dielectric loaded waveguide were investigated in Sec. V. It was found that a strong mode coupling between the vacuum dielectric waveguide and beam streaming modes occurs in a range of physical parameters, exhibiting possibilities of a strong Cherenkov radiation. The maximum growth rate of instability is order of the beam plasma frequency. In this regard, the Cherenkov radiation can

also be an effective means to produce intense high power microwave.

The wavelength of the microwave radiation can be less than a centimeter.

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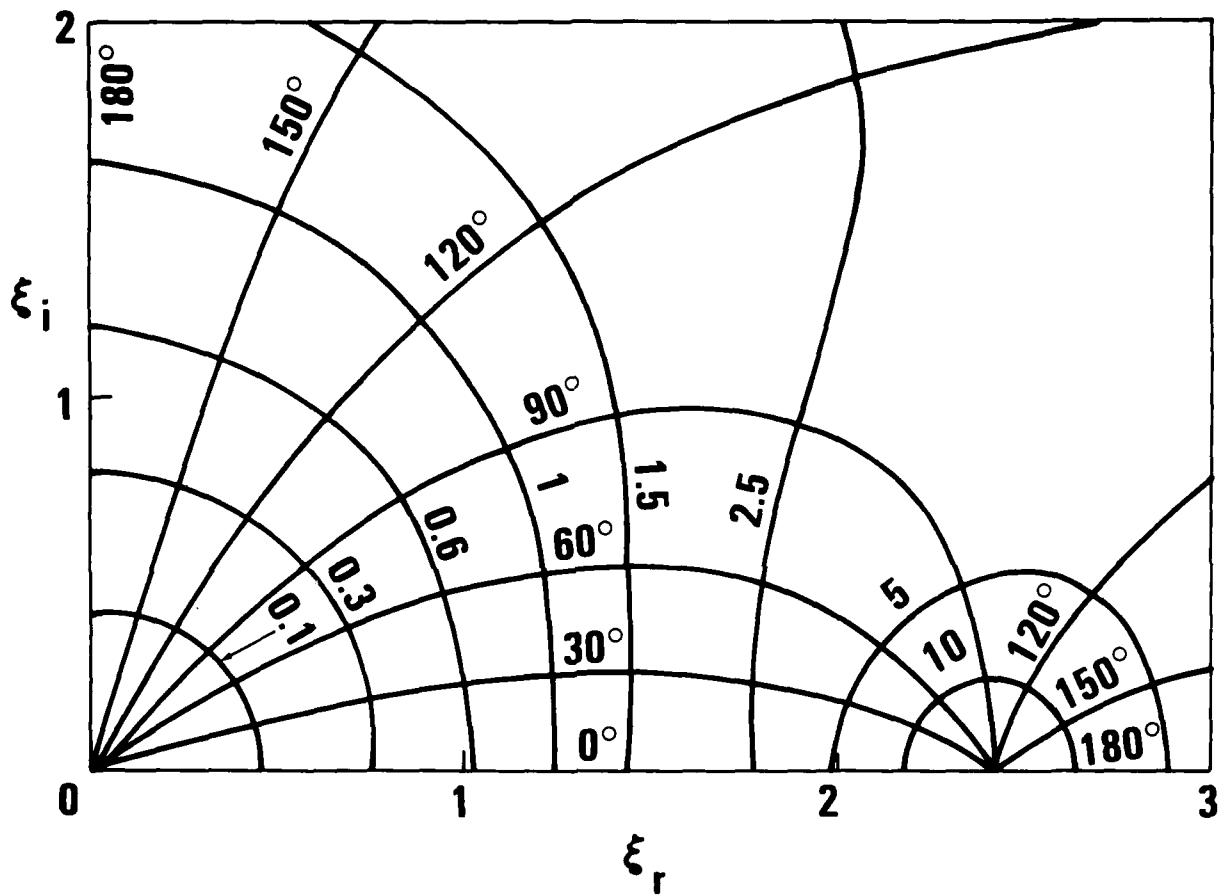


FIGURE 1 CONTOURS OF CONSTANT PHASE ANGLE  $\phi$  AND MODULUS  $f$  [EQ. (18)]  
IN THE COMPLEX PLANE  $\xi = (\xi_r, \xi_i)$  FOR  $n = 1$  RADIAL MODE NUMBER.

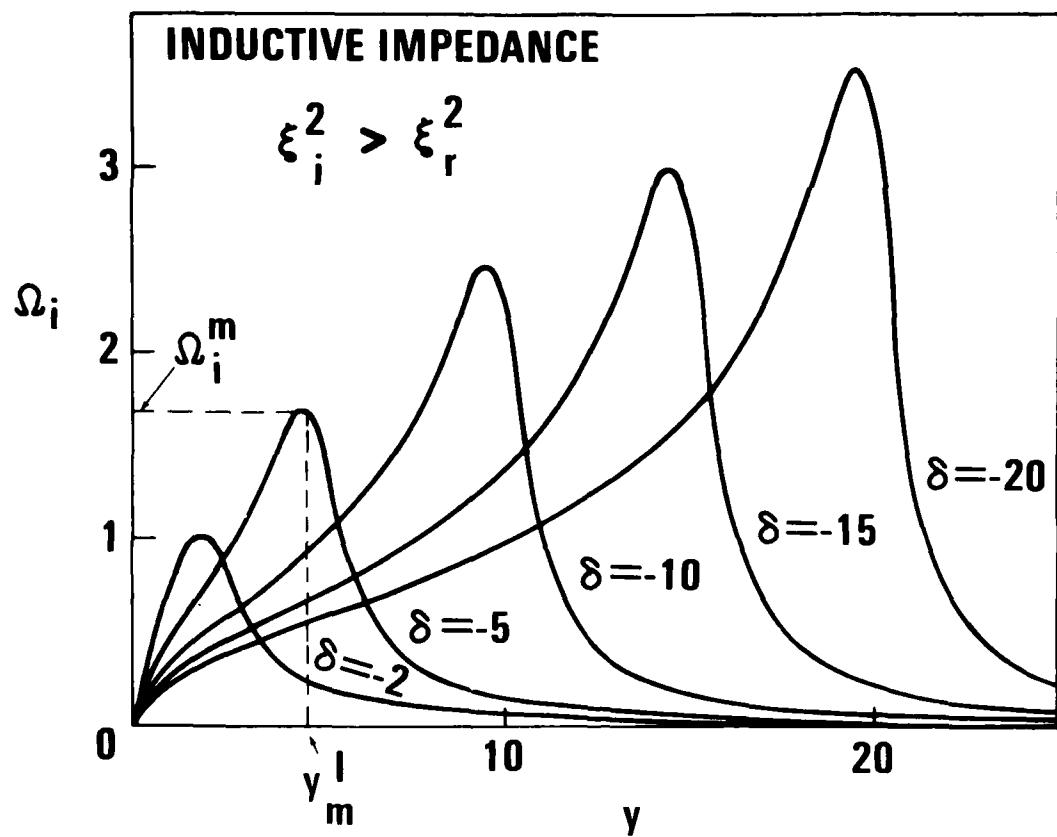


FIGURE 2 PLOT OF THE NORMALIZED GROWTH RATE  $\Omega_i$  VERSUS  $\gamma$  [EQ. (27)]  
FOR  $\xi_i^2 > \xi_r^2$  AND SEVERAL VALUES OF  $\delta$ .

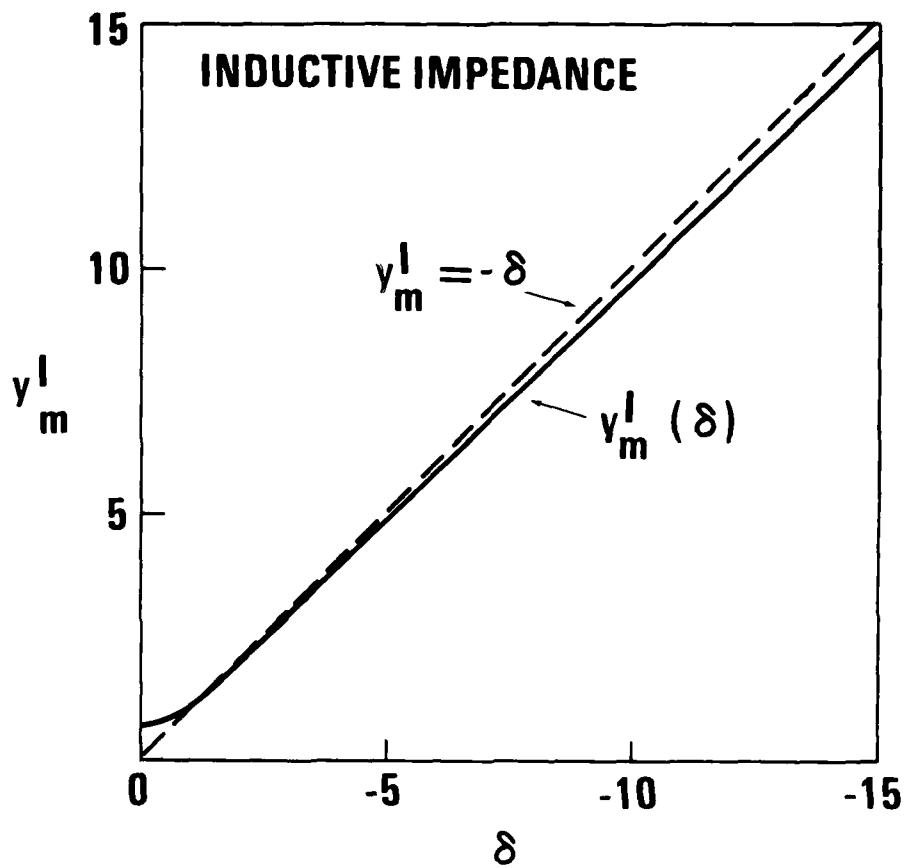


FIGURE 3 PLOT OF THE PARAMETER  $y_m^I$  VERSUS  $\delta$  (SOLID CURVE) OBTAINED FROM EQ. (27) FOR  $\xi_i^2 > \xi_r^2$ . THE DASHED STRAIGHT LINE IS  $y_m^I = -\delta$ .

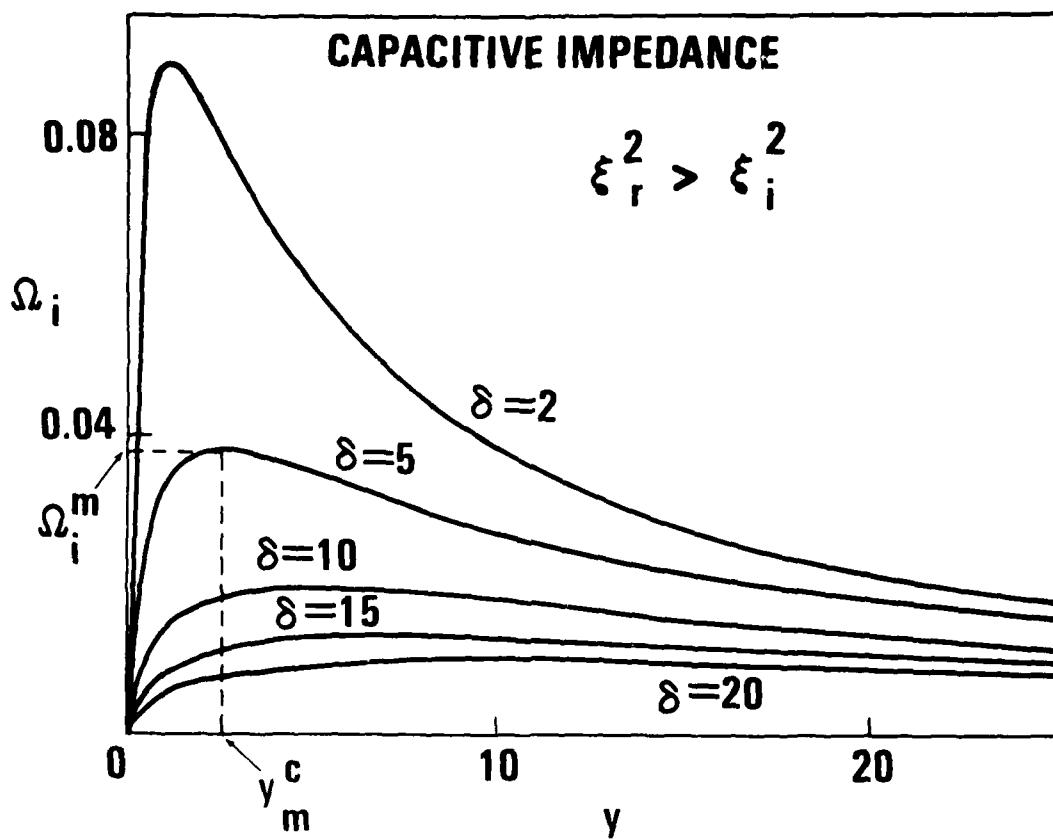


FIGURE 4 PLOT OF THE NORMALIZED GROWTH RATE  $\Omega_i$  VERSUS  $\gamma$  [EQ. (27)]  
FOR  $\xi_r^2 > \xi_i^2$  AND SEVERAL VALUES OF  $\delta$ .

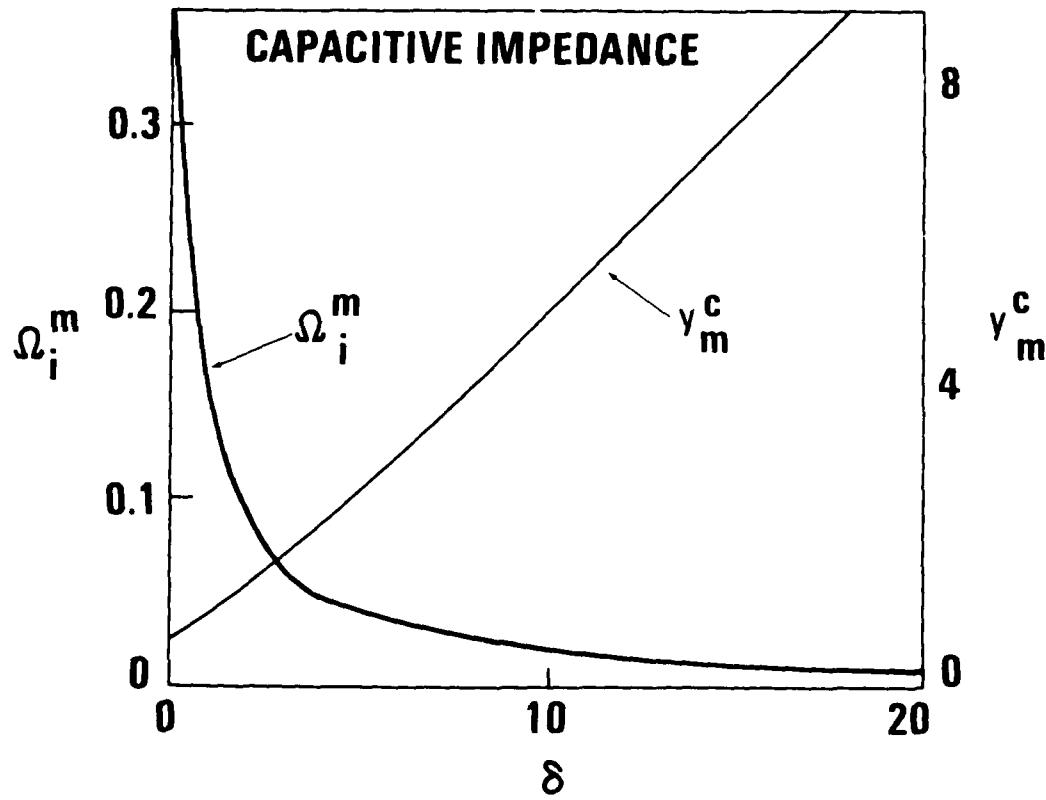


FIGURE 5 PLOT OF THE NORMALIZED MAXIMUM GROWTH RATE  $\Omega_i^m$  AND CORRESPONDING  $y_m^c$  VERSUS  $\delta$  [EQ. (27)] FOR  $\xi_r^2 > \xi_i^2$ .

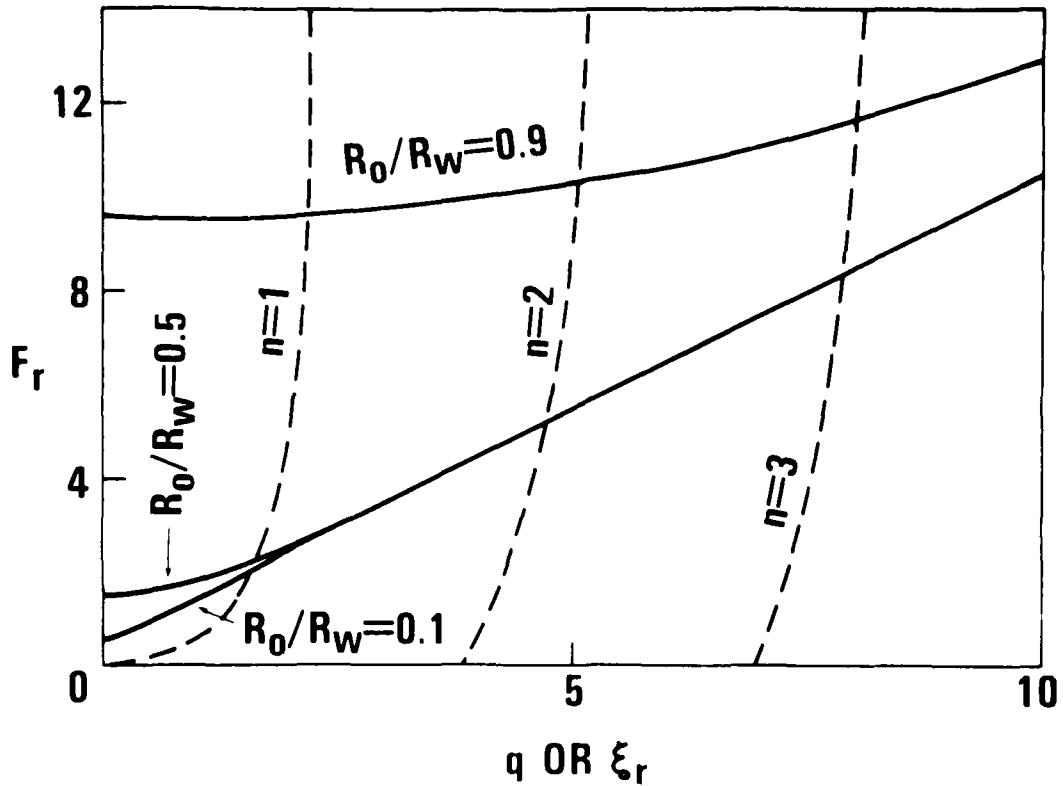
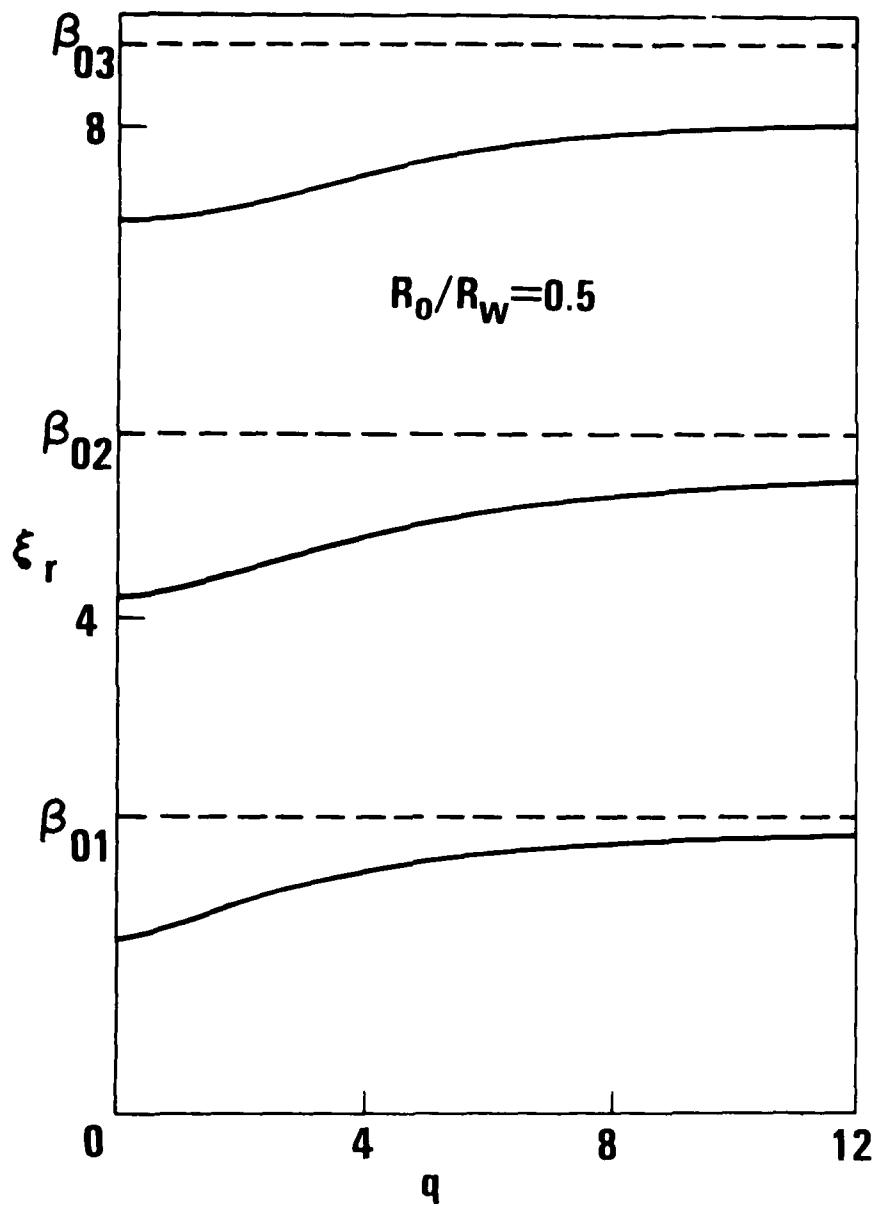


FIGURE 6 PLOT OF THE ADMITTANCE  $F = F_r$  VERSUS  $\xi = \xi_r$  (DASHED CURVES) OBTAINED FROM EQ. (18) AND  $F_r$  VERSUS  $q$  (SOLID CURVES) OBTAINED FROM EQ. (38) FOR  $R_0/R_W = 0.1, 0.5$ , AND  $0.9$ . THE HORIZONTAL SCALE REPRESENTS BOTH  $\xi_r$  AND  $q$ .

FIGURE 7 PLOT OF  $\xi_r$  VERSUS  $q$  DETERMINED FROM FIGURE 6 FOR  $R_0/R_w = 0.5$ .

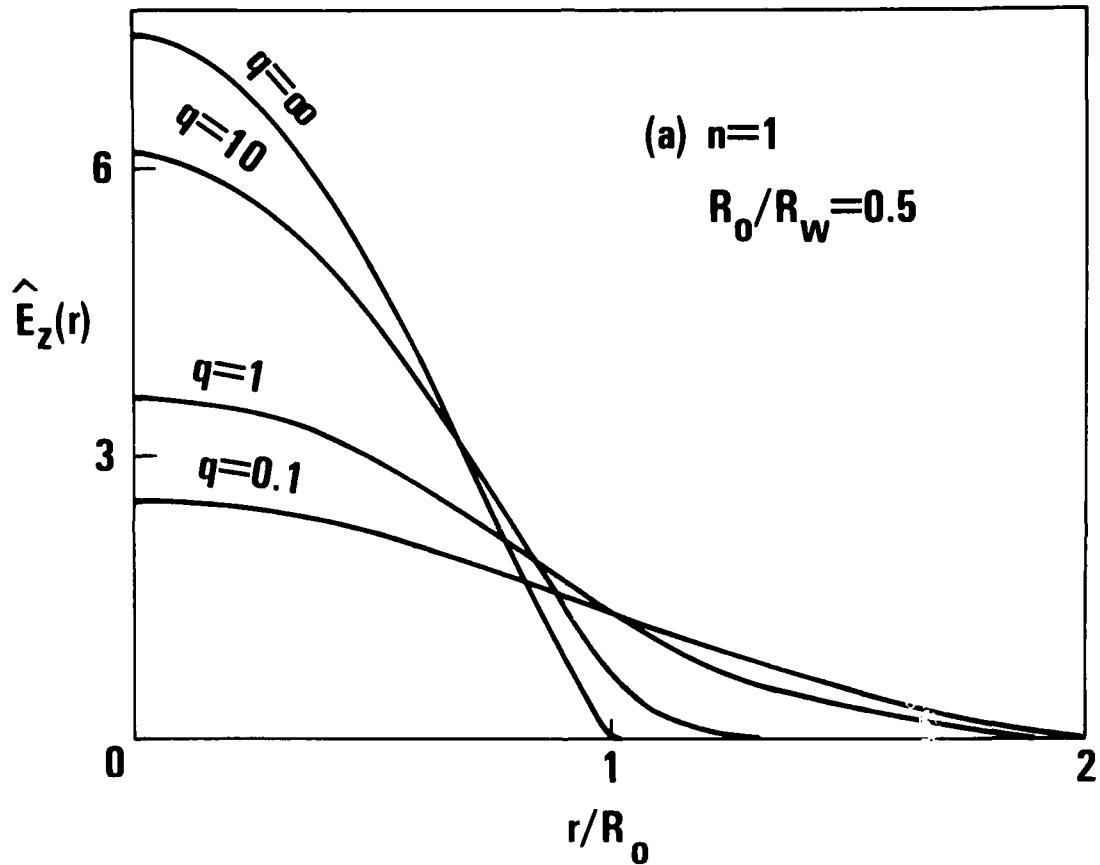


FIGURE 8a PLOT OF THE EIGENFUNCTION  $\hat{E}_z(r)$  VERSUS  $r/R_0$  [EQ. (17)] FOR  $n = 1$ ,  $R_0/R_w = 0.5$ , AND SEVERAL VALUES OF  $q$ .

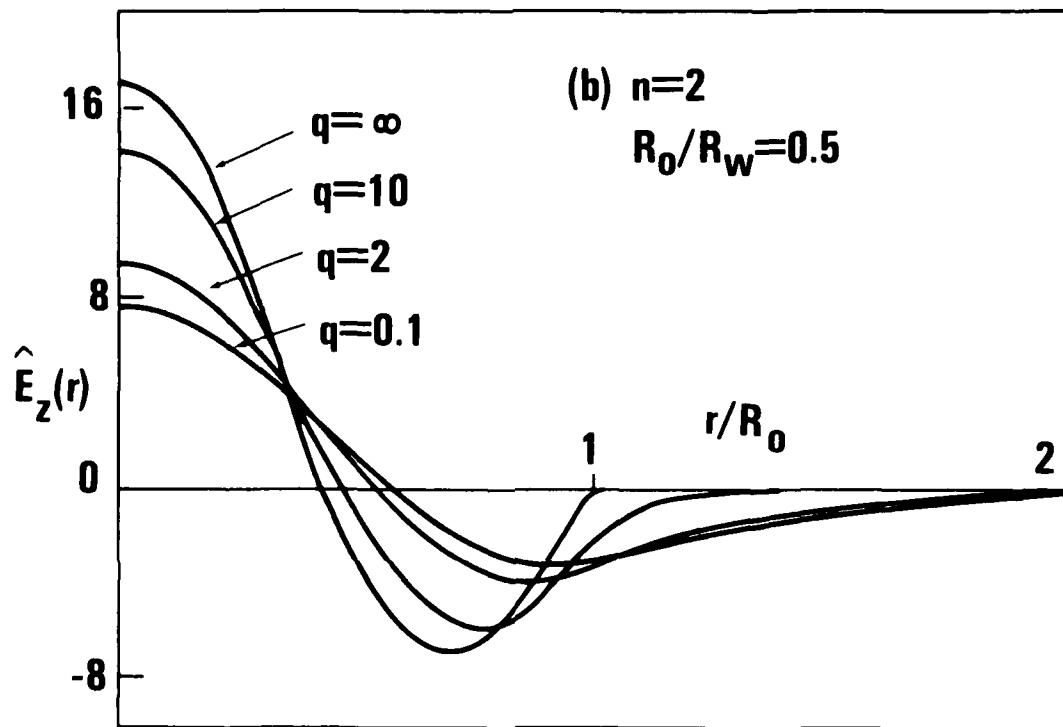


FIGURE 8b PLOT OF THE EIGENFUNCTION  $\hat{E}_z(r)$  VERSUS  $r/R_0$  [EQ. (17)] FOR  $n = 2$ ,  $R_0/R_w = 0.5$ , AND SEVERAL VALUES OF  $q$ .

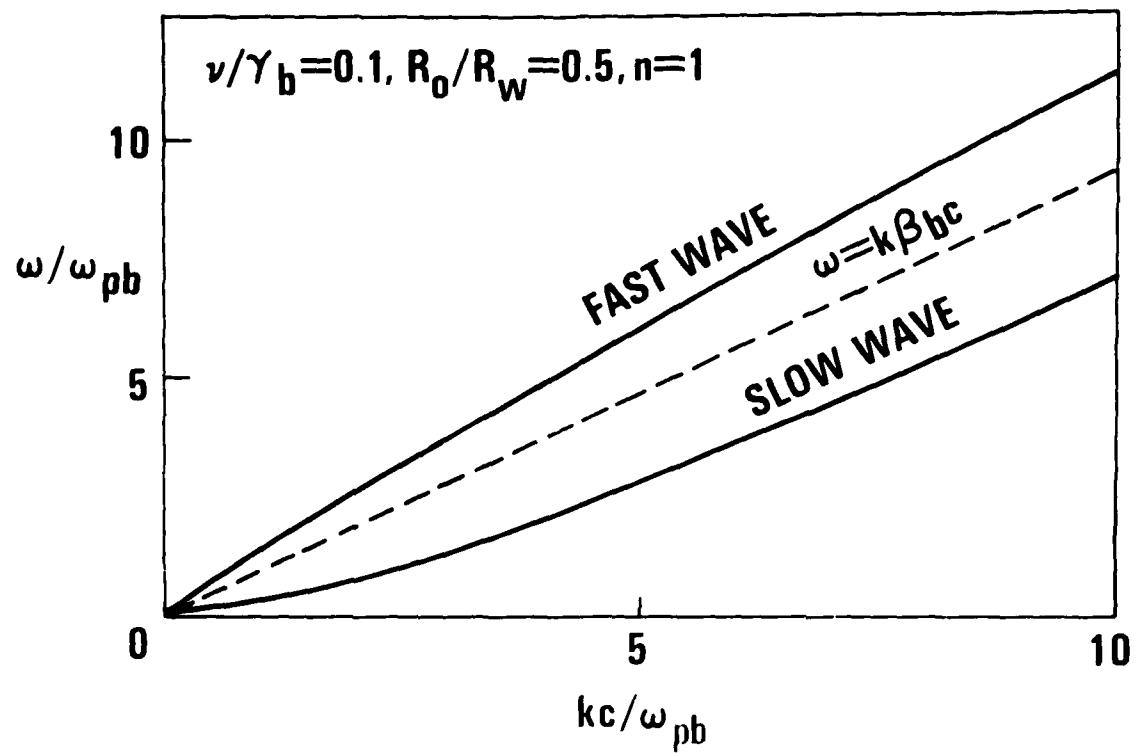


FIGURE 9 PLOT OF THE DISPERSION CURVE IN THE PARAMETER SPACE ( $\omega, k$ ) FOR  $n = 1$ ,  $R_0/R_w = 0.5$ ,  $\gamma_b = 3$ , AND  $\nu/\gamma_b = 0.1$ . THE DASHED STRAIGHT LINE  $\omega = k\beta_b c$  IS THE FREE-STREAMING MODE.

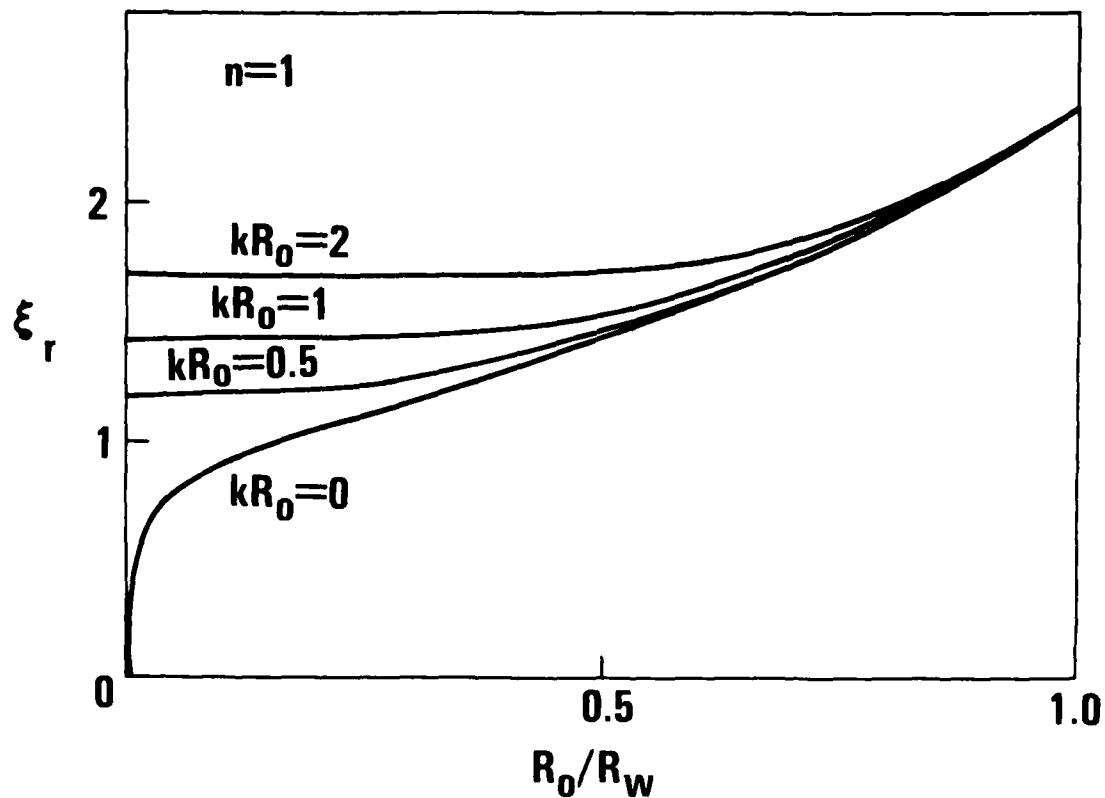


FIGURE 10 PLOT OF  $\xi_r$  [REQUIRED IN EQ. (42)] VERSUS  $R_0/R_w$  FOR  $n = 1$  AND SEVERAL VALUES OF  $kR_0$ .

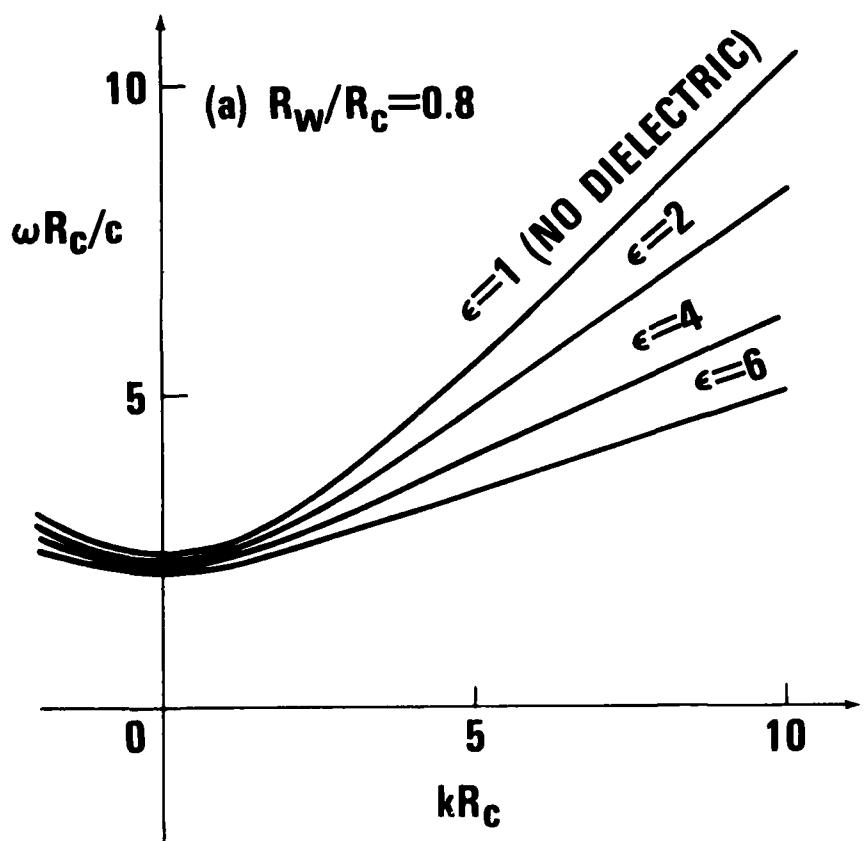


FIGURE 11a PLOT OF THE VACUUM TM MODE DISPERSION RELATION IN THE PARAMETER SPACE ( $\omega, k$ ) OBTAINED FROM Eqs. (48), (50), AND (51) FOR  $R_w/R_c = 0.8$  AND SEVERAL VALUES OF THE DIELECTRIC CONSTANT  $\epsilon$ .

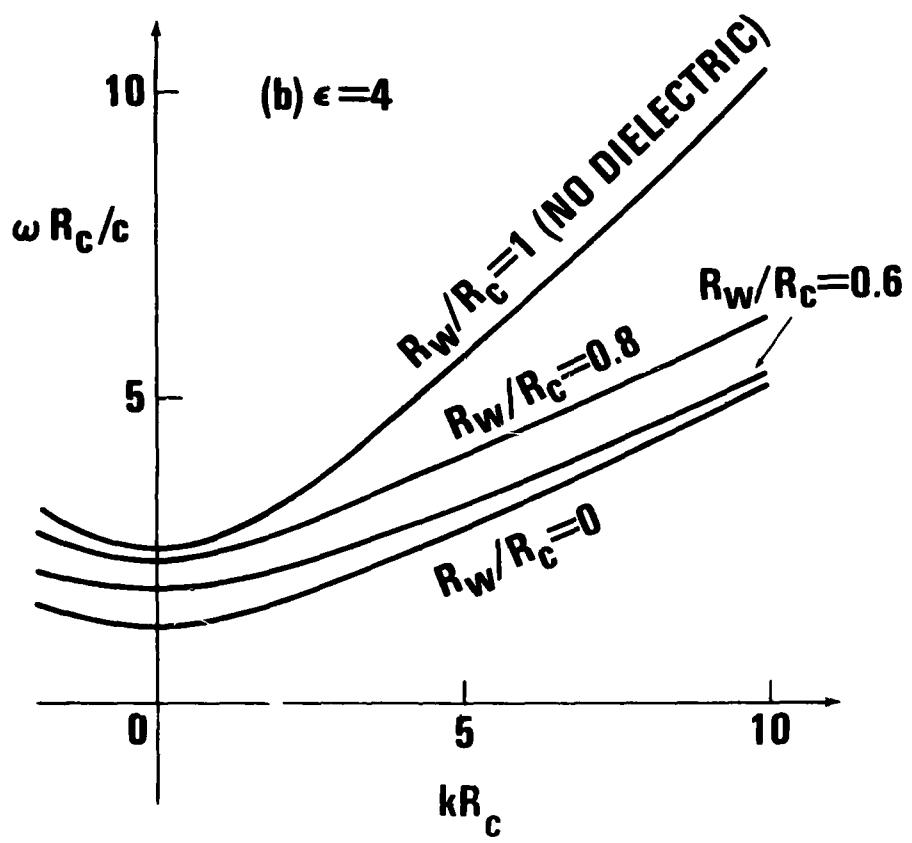


FIGURE 11b PLOT OF THE VACUUM TM MODE DISPERSION RELATION IN THE PARAMETER SPACE ( $\omega, k$ ) OBTAINED FROM Eqs. (48), (50), AND (51) FOR  $\epsilon = 4$  AND SEVERAL VALUES OF THE RATIO  $R_w/R_c$ .

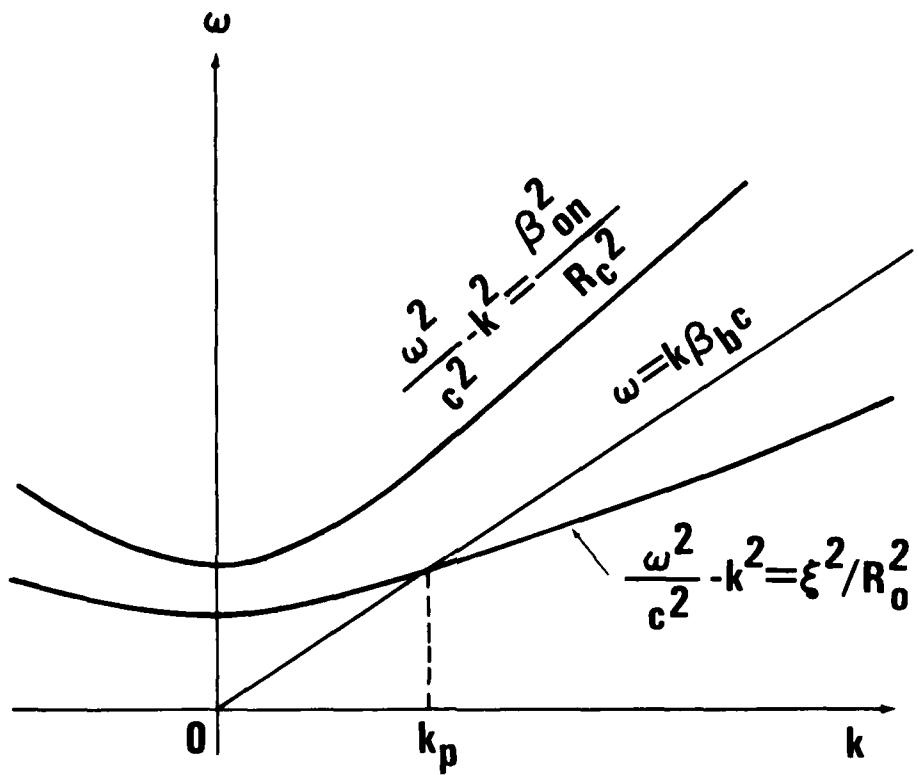


FIGURE 12 SKETCH OF  $\omega = (k^2 c^2 + \beta_{0n}^2 c^2 / R^2)^{1/2}$  VERSUS  $k$  (CORRESPONDING TO PERFECTLY CONDUCTING WAVEGUIDE) AND  $\omega = (k^2 c^2 + \xi_0^2 c^2 / R_0^2)^{1/2}$  VERSUS  $k$  (CORRESPONDING TO AN ARBITRARY WALL IMPEDANCE  $Z$ ). THE STRAIGHT LINE  $\omega = k\beta_b c$  IS THE FREE-STREAMING MODE.

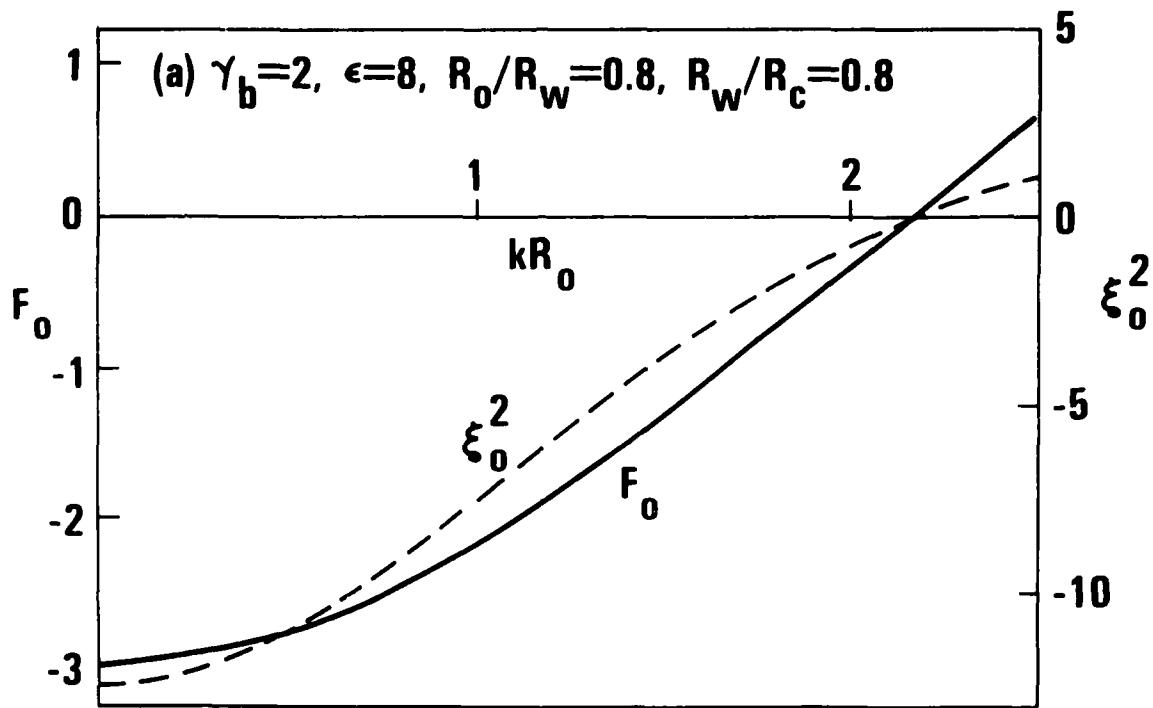


FIGURE 13a PLOTS OF  $F_0$  (SOLID CURVE) AND  $\xi_0^2$  (DASHED CURVE), [Eqs. (57), (58), AND (61)] FOR  $\gamma_b = 2, \epsilon = 8, R_0/R_w = 0.8, R_w/R_c = 0.8$ , AND  $v = 0.0025$ . THE REAL OSCILLATION FREQUENCY IS OBTAINED FOR  $\Delta = 0$ .

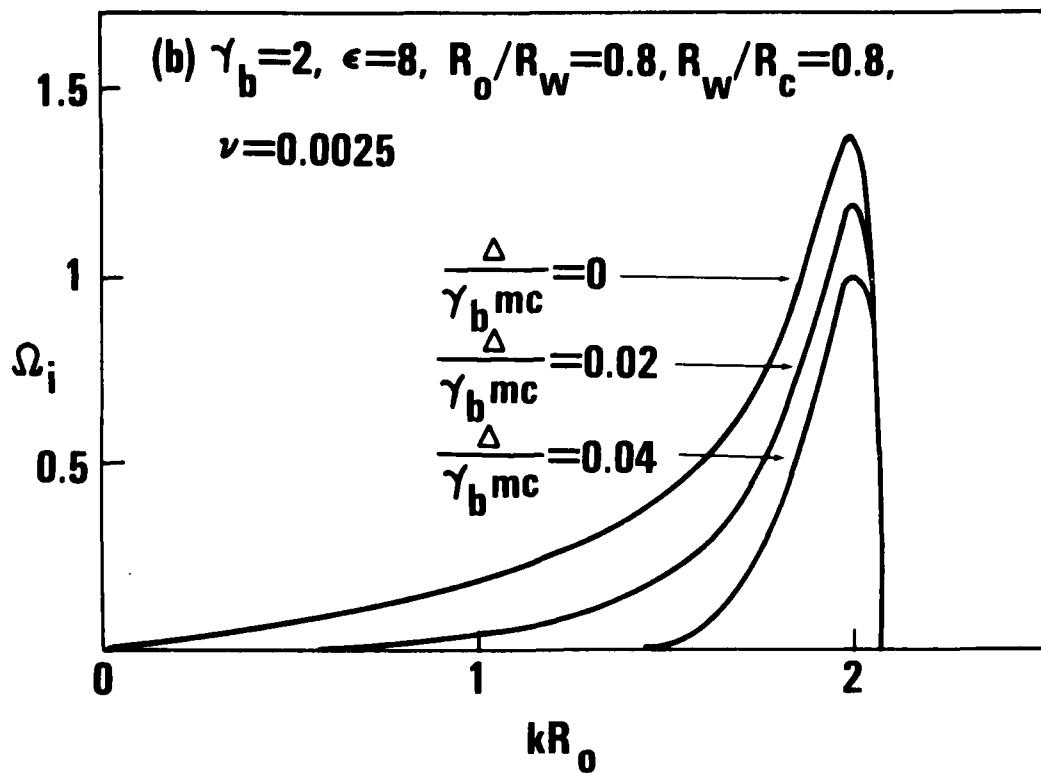


FIGURE 13b PLOTS OF THE NORMALIZED GROWTH RATE  $\Omega_i = 1m\Omega$  [Eqs. (57), (58), AND (61)] FOR  $\gamma_b = 2$ ,  $\epsilon = 8$ ,  $R_0/R_w = 0.8$ ,  $R_w/R_c = 0.8$ , AND  $\nu = 0.0025$ . THE REAL OSCILLATION FREQUENCY IS OBTAINED FOR  $\Delta = 0$ .

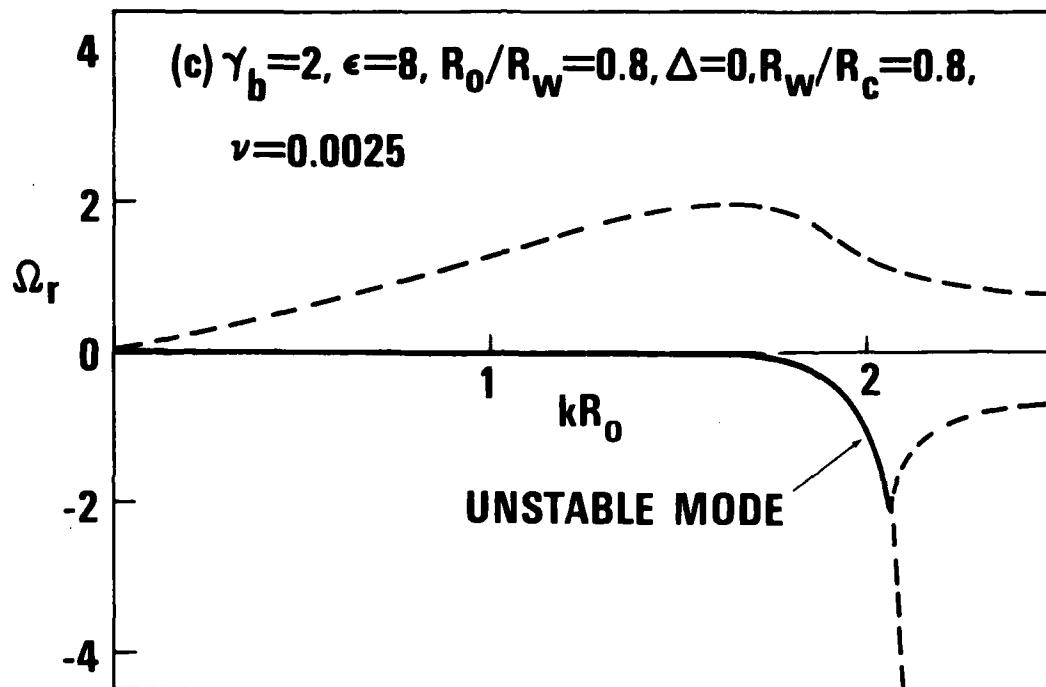


FIGURE 13c PLOTS OF DOPPLER-SHIFTED REAL OSCILLATION FREQUENCY  $\Omega_r = \text{Re}\Omega$  VERSUS  $kR_0$  [Eqs. (57), (58), AND (61)] FOR  $\gamma_b = 2, \epsilon = 8, R_0/R_w = 0.8, R_w/R_c = 0.8$ , AND  $\nu = 0.0025$ . THE REAL OSCILLATION FREQUENCY IS OBTAINED FOR  $\Delta = 0$ .

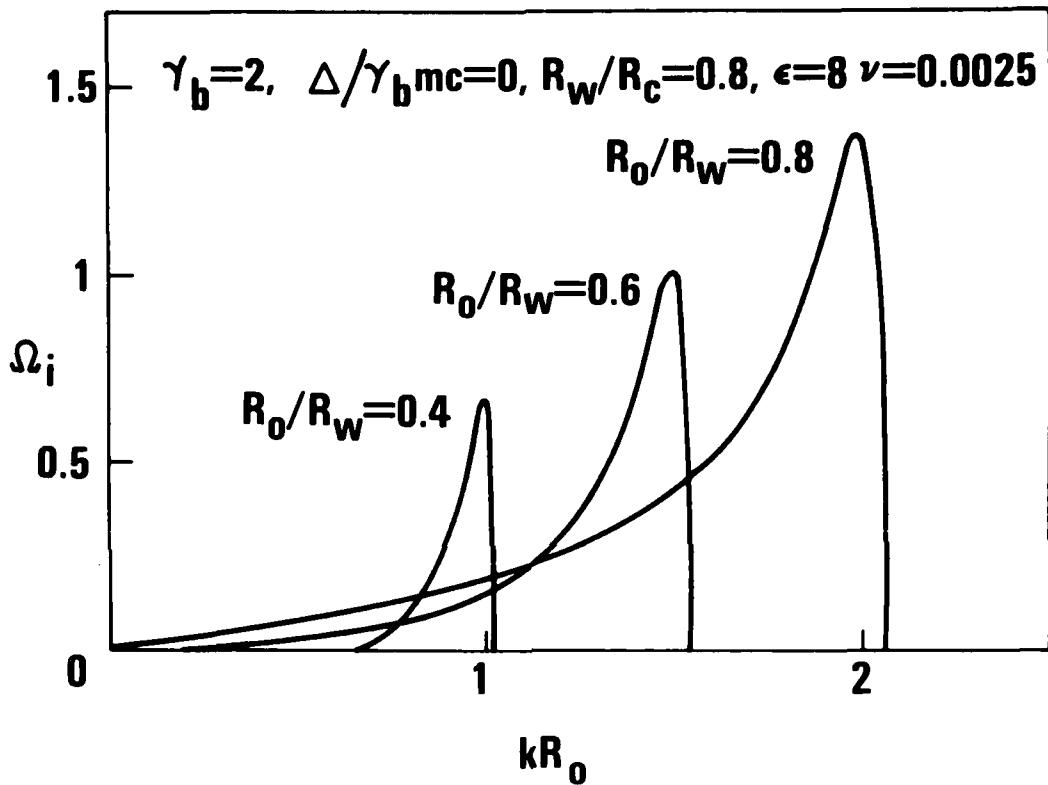


FIGURE 14 PLOT OF THE NORMALIZED GROWTH RATE  $\Omega_i = 1m\Omega$  VERSUS  $kR_0$  [EQ. (61)] FOR  $\Delta = 0$ , SEVERAL VALUES OF  $R_0/R_w$ , AND PARAMETERS OTHERWISE IDENTICAL TO FIGURE 13.

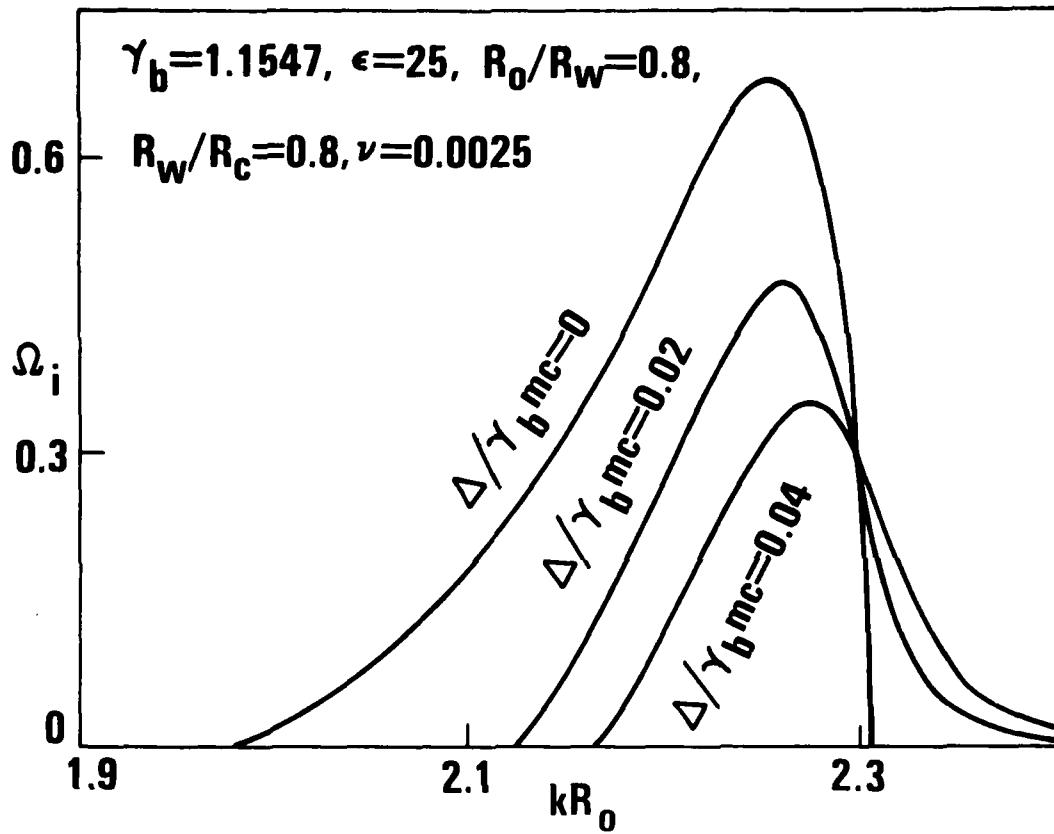


FIGURE 15 PLOT OF THE NORMALIZED GROWTH RATE VERSUS  $kR_0$  [EQ. (61)] FOR  $\gamma_b = 1.1547, \epsilon = 25$ , AND PARAMETERS OTHERWISE IDENTICAL TO FIGURE 13.

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